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BRIEF COMMUNICATION

EFFECT OF VISCOSITY ON DROP BREAKUP†

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1. INTRODUCTION

The subject of breakup of liquid drops has important implications in a variety of fields, namely, but not specifically, meteorology, aerosol and hydrosol science, and emulsification processes. As a result, an extensive literature, covering a wide spectrum of related theoretical and experimental works, has been generated over the years (Clift *et al.* 1978; Pilch & Erdman 1987; Hsiang & Faeth 1992 and references therein). A major goal of many of these studies has been to provide information on and explanations of how the external flow conditions affect the critical Weber number, We_{cr}, a quantity representing the minimum force or energy required to cause primary breakup of the liquid drop.

Numerous experimental investigations have dealt with the breakup of liquid drops in gases due to shocks. Here, in general, the Weber number, We, is defined as

$$We \equiv \frac{\rho_G U^2 d_0}{\sigma},$$ [1]

where ρ_G is the gas density, σ is the interfacial tension and U and d_0 are the initial relative gas velocity and drop diameter, respectively. An important finding in many of these works has been the existence of several regimes or modes of drop disintegration. These regimes, which are termed "bag", "multimode" and "shear", have been observed to occur at different levels of We. For a comprehensive description and analysis of these regimes, the reader is referred to Hsiang & Faeth (1992).

Of further interest to these studies has been the effect of drop viscosity on the breakup Weber number, We_{br}, for the different modes. All works dealing with this effect concluded that an increase in the drop viscosity leads to a rise in We_{br}; the reason being that an additional amount of energy is needed to overcome the internal viscous dissipation, induced by drop deformation and mixing within the liquid phase.

To theoretically predict We_{br} in the presence of liquid-phase viscosity, the conventional approaches have, in one way or another, implemented the momentum equations (Kitscha & Kocamustafaogullari 1989; Tarnogrodzki 1993). The intention here, however, is different in that we focus on using energy-related arguments to provide a semi-empirical correlation relating We_{br} with viscosity and other properties. As we shall demonstrate, the resulting correlation could be applicable to a wide range of We, corresponding to the various modes of drop breakup by shocks.

2. PROBLEM FORMULATION

Following an energy approach similar to one taken earlier (Cohen 1991), we assume that the energy, E_{Tbr}, needed to break a viscous drop having an initial diameter d_0 equals the breakup energy

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required in the limit of zero viscosity, i.e. $E_{Tbr}(\mu_L \to 0)$, where μ_L is the viscosity of the liquid drop, raised by an amount equivalent to the viscous energy dissipated within the drop during deformation and breakup. Letting E_v represent this viscous dissipation, the above can then be written as

$$E_{Tbr} = E_{Tbr}(\mu_L \to 0) + E_v. \qquad [2]$$

In other words, the increase in We_{br}, when viscosity is present, is being attributed to the extra energy dissipated within the liquid drop. Since dissipation is always positive, one should then expect that an extra quantity of energy, in addition to the critical energy at zero viscosity, is to be supplied in order for the drop to burst. This is consistent with the observed rise in the drop We_{br}, when the liquid-phase viscosity is not zero.

Dividing both sides of [2] by the initial surface energy, $\pi \sigma d_0^2$, yields

$$\frac{E_{Tbr}}{\pi \sigma d_0^2} = \frac{E_{Tbr}(\mu_L \to 0)}{\pi \sigma d_0^2} + \frac{E_v}{\pi \sigma d_0^2}. \qquad [3]$$

Obviously, the terms appearing above are dimensionless groups, two of which involve the "impact" energy absorbed by the drop, divided by the initial surface energy. The breakup energy term, E_{Tbr}, which can be expressed as the kinetic energy imparted to the drop by the gas (Cohen 1991), i.e.

$$E_{Tbr} = \frac{\pi d_0^3 \rho_G}{6} \frac{U^2}{2}, \qquad [4]$$

may also be viewed as the pressure differential induced by the external flow, $\rho_G U^2/2$, acting on the drop, multiplied by the volume of the drop, $\pi d_0^3/6$. Combining the above with [1] yields

$$We_{br}(\mu_L \to 0) = \frac{12 E_{Tbr}(\mu_L \to 0)}{\pi \sigma d_0^2} \qquad [5a]$$

and

$$We_{br} = \frac{12 E_{Tbr}}{\pi \sigma d_0^2}, \qquad [5b]$$

where $We_{br}(\mu_L \to 0)$ is the breakup Weber number in the limit of zero drop viscosity, and We_{br} is that of a viscous drop.

Based on dimensional and physical grounds, we express E_v as

$$E_v \approx \mu_L V_{mix} d_0^2 \qquad [6]$$

for a Newtonian liquid. Here, V_{mix} is an effective internal "mixing rate", which we have introduced as a lumped parameter to reflect the mixing motions within the drop prior to breakup. The proportionality constant of order unity, which is supposed to appear in [6], has, for convenience, been embedded in V_{mix}. Moreover, while V_{mix} would most likely depend on the type of external flow, be it couette, parallel, hyperbolic etc., as well as the magnitude of the We, we plan to treat it as empirical, and seek to extract its behavior from the available experimental data.

Substituting [5] and [6] into [3] yields

$$\frac{We_{br}}{\dfrac{We_{br}(Nv \to 0)}{1 + K^* Nv}} = 1 \qquad [7]$$

after some rearrangement. Here, K^* is simply

$$K^* \equiv \frac{\dfrac{12}{\pi}}{We_{br}(Nv \to 0)} V_{mix} \sqrt{\frac{\rho_L d_0}{\sigma}} \qquad [8]$$

and Nv, which is the viscosity or Ohnesorge number, is defined as

$$Nv \equiv \frac{\mu_L}{\sqrt{\rho_L \sigma d_0}}, \qquad [9]$$

with ρ_L being the liquid-drop density. By containing V_{mix}, therefore, K^* may, in general, be expressible as a function of the several parameters involved; i.e.

$$K^* = K^*\left(We_{br}, We_{br}(Nv \to 0), Nv, \frac{\rho_L}{\rho_G}\right). \qquad [10]$$

Assessing the behavior of K^* from the available experimental data constitutes the focal point of the next section.

3. ANALYSIS AND CONCLUSIONS

It is now important to emphasize that, in order to incorporate the effects of the drop viscosity, μ_L, into a more general theoretical framework, Hinze (1955) proposed the following relation:

$$\frac{We_{br}}{\dfrac{We_{br}(Nv \to 0)}{1 + fn(Nv)}} = 1, \qquad [11]$$

where $fn(Nv)$ is a function that depends on Nv in such a way that it approaches zero as $Nv \to 0$. How this function varies with Nv has been a topic of investigation, and, as a result, certain power-law type expressions, one being (Brodkey 1969)

$$We_{br} = 12(1 + 1.077Nv^{1.6}), \qquad [12]$$

which is for bag breakup [where, for this case, $We_{br}(Nv \to 0) \approx 12$], have been proposed based strictly on empiricism. Nonetheless, in view of the similarity between [7] and [11], it seems that an attempt to deduce this functional dependence using the energy-conservation arguments discussed in the previous section should be worthwhile.

Our objective now is to determine the behavior of K^*, as laid down in [10], that would make [11] consistent with [7]. Obviously, based on a comparison of the two equations, the following form for K^*—i.e. one that would make the denominator of [7] a function of Nv only—could be recommended:

$$K^* = K^*(Nv) \text{ or constant}; \qquad [13]$$

suggesting that K^* either depends only on Nv, or, more simply, it is a numerical constant.

By virtue of [7], the dependence of K^* on Nv can now be deduced easily from We_{br} vs Nv data, plotted as $\{We_{br}/We_{br}(Nv \to 0) - 1\}$ vs Nv on a log–log scale. Applying this to the data available to us [figure 4 of Hinze (1955), figure 2 of Pilch & Erdman (1987) and figure 1 of Hsiang & Faeth (1992)] gave rise to plots quite similar to that in figure 1 of this paper.
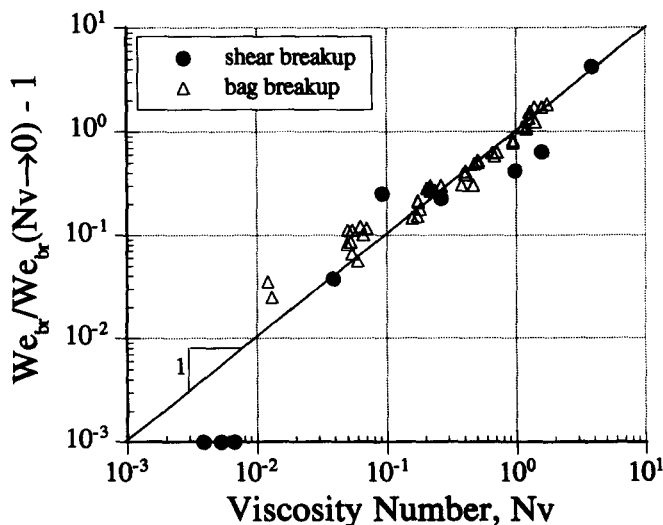

Figure 1. Plot of $\{We_{br}/We_{br}(Nv \to 0) - 1\}$ vs Nv: ●, shear-mode breakup data obtained from Hsiang & Faeth (1992); △, data of Hinze (1955).
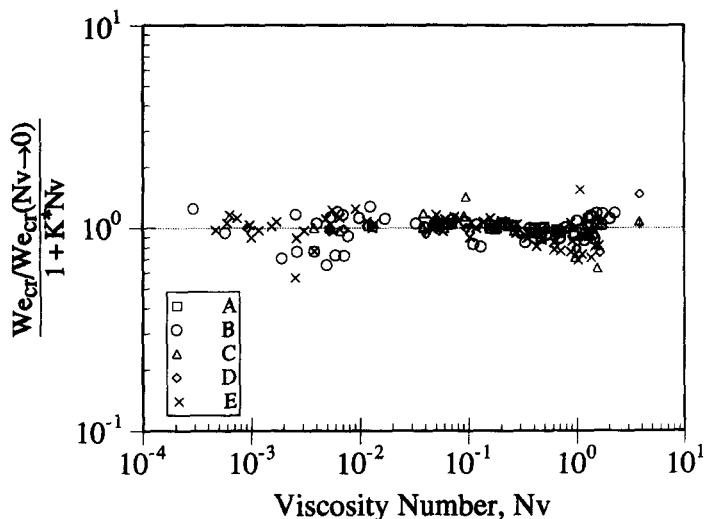
Figure 2. The left-hand side of [7] plotted vs Nv. Sources for the data points A–E, along with other pertinent information, are given in table 1.

We should mention, however, that all the data we examined, except for Hinze's which is included in figure 1, display a significant amount of "noise" at very low values of Nv when plotted this way. The reason for this may be that determinations of breakup regimes are rarely highly accurate, a limitation that deems the scatter at small values of Nv reasonable. Nonetheless, for almost all the data used here, the noise disappears at higher Nv, i.e. Nv $\gtrsim$ 0.01, and linear behaviors with slopes very close to unity do indeed emerge [see, for example, the shear breakup mode, which is also shown in figure 1]. This, along with the fact that all $\{We_{br}/We_{br}(Nv \to 0) - 1\}$ values must approach zero as Nv$\to$0, could suggest that K^* might very well be a constant, independent of Nv.

With this in mind, in figure 2 we have replotted the data according to the formulation of [7]. The liquid–liquid data present in figure 2 of Pilch & Erdman (1987), however, have been excluded in order to maintain the density ratio at a level pertinent to liquid-in-gas systems.

It should be pointed out that for every data set, each corresponding to a specific regime of breakup, we found it necessary to adjust the constant K^* slightly in order to force the left-hand side of [7] to equal, or at least come close to unity, as required. The dependence of K^* on $We_{br}(Nv \to 0)$ is displayed in figure 3, and its numerical values, along with other relevant information, are supplied in table 1. Overall, K^* averages at 1.4, which is of the order of unity, has a standard deviation of about 0.38, and displays a trend that leans more towards constant behavior. This further supports our previous contention that K^*, as given by [13], is either a numerical constant, or, at most, dependent only on Nv. For curiosity, the value of K^* for the liquid–liquid data in figure 2 of Pilch & Erdman (1987) was also computed. This was found to be about 1.3, which lies well within the neighborhood.

Based on the above, therefore, the semi-empirical formulation of [7] can be recast into

$$\frac{We_{br}}{We_{br}(Nv \to 0)} \approx 1 + K^* Nv \quad \text{(where } 1.0 \leqslant K^* \leqslant 1.8\text{).} \tag{14}$$

This is expected to hold for $10 < We_{br}(Nv \to 0) < 10^2$, covering the entire range of the three modes of shock-induced drop-breakup regimes considered here. As to which is better, [14] or [12], where Nv carried a power different from 1, one could argue that it might be a matter of personal choice since, based strictly on visual judgment, both seem to represent the given experimental data reasonably well. Equation [14], notwithstanding, may be more appealing simply because (i) it has been derived based on the energy-conservation principle and (ii) its application is not limited to a single breakup mode, whereas [12] is applicable only to bag breakup.
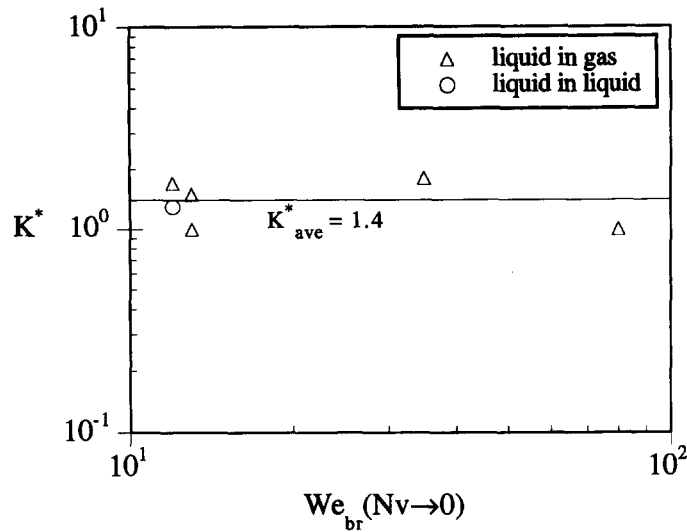
Figure 3. K^* plotted vs the parameter $We_{br}(Nv \to 0)$: $\bigcirc$, the value for K^* extracted from the liquid–liquid data in figure 2 of Pilch & Erdman (1987); $\triangle$, liquid–gas data. The horizontal line is the average K^*.

Table 1. Description of the data in figure 2

Data	Source	Breakup mode	$We_{br}(Nv \to 0)$	K^*
A	Figure 4 of Hinze (1955)	Bag	13	1.0
B	Figure 1 of Hsiang & Faeth (1992)	Bag	13	1.5
C	Figure 1 of Hsiang & Faeth (1992)	Shear	80	1.0
D	Figure 1 of Hsiang & Faeth (1992)	Multimode	35	1.8
E	Figure 2 of Pilch & Erdman (1987)	Bag	12	1.7

Moreover, substituting $K^* = 1.4$ into [8] and rearranging, provides the following dimensionless group involving V_{mix}:

$$V_{mix} \sqrt{\frac{\rho_L d_0}{\sigma}} \approx 0.366 \, We_{br}(Nv \to 0); \tag{15}$$

which should remain valid for density ratios, ρ_L/ρ_G, relevant to liquid–gas systems. With the dynamics of the drop fragmentation manifested within the single parameter V_{mix}, the fact that it is not influenced by viscosity could suggest that the breakup mode belonging to any given regime remains independent of the liquid viscosity, and, thereby, it is preserved. In other words, each mode of breakup maintains its form, even as the liquid viscosity is changed. This seems consistent with experiments [see, for instance, figure 1 of Hsiang & Faeth (1992)], whereby upon selecting a certain breakup regime, which would be describable by its own $We_{be}(Nv \to 0)$, and beginning at a viscosity number close to zero, a gradual increase in Nv raises only the value of We_{br} and, apparently, does not alter the mode of breakup. This, clearly, is indicative of self-preservation. Also, realizing that We_{br} is proportional to $\rho_G U^2$, it is then easy to conclude from [15] that the mixing rate within the drop is related directly to the external dynamic pressure field in the limit $Nv \to 0$.

Lastly, what this work offers is a correlation, deduced by elementary energy-based arguments, relating We_{br} to Nv. The simple idea that much of the dynamics of drop breakup could be lumped into a single parameter, V_{mix}, may call for a more definitive study of the subject from this perspective.

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